FLAT COILS

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1. Abstract. Air core coils can be made in a flat configuration with thickness of about two diameters of the conducting strand plus insulation. They can be wound of wire (WPI's: wound planar inductors) or made on printed circuit boards (PPI's: printed planar inductors). The area of these flat coils for a given inductance is equal to the projected area of the comparable helix plus the area of one turn of that helix. Advantages over conventional helical coils include greatly reduced bulk, increased mechanical and electrical stability because of flat supporting mounting, high coefficients of coupling, the possibility of making them by etched circuit methods. Distributed capacitances are higher and Q's slightly lower than for comparable helixes.

2. Flat Coils

Practically all air core inductances of the past one hundred years of electrical engineering have been relatively bulky three-dimensional devices. Work described in this paper shows that these inductances can be made in planar form with advantages for many applications.

Fig. 1 is a photograph of several experimental flat coils and one helix. Table I shows some of their characteristics.

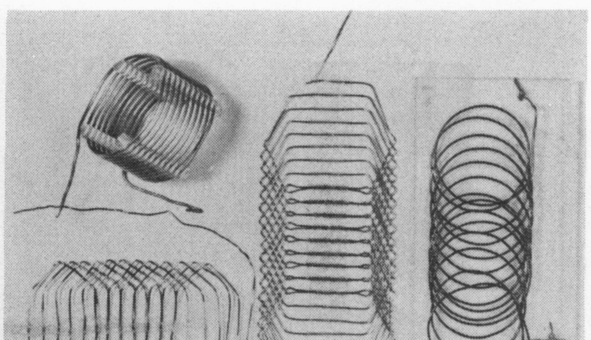

Fig. 1: An air core helix and various flat coils. Table I gives data.

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The flat coil with circular turns in Fig. 1 shows its derivation from the helix by pushing the turns of the helix over until they all lie substantially in the same place. The shape of each turn is preserved, but its position with respect to the other turns is altered. Such a coil is designated WPI for wound planar inductor.[1] We use the term "spacing" in the helical coil to designate the distance between adjacent turns, center-to-center. We will use the same term in the flat coil to designate the displacement between adjacent turns along the long axis.

The remarkable thing about the flat coil is that it still has, within a few per cent, the same inductance as the helix (given the same number of turns, radius of turn, and spacing between turns).

This result may appear anomalous but a simple theoretical justification is given below. The result is established by numerous experimental measurements.

These flat coils may be wound with any turn shape desired. Hexagonal (as shown in Fig. 1) and lozenge-shaped turns overcome the circular turns' bunching effect at top and bottom.

The turns must of course be insulated from each other at the crossings. Enamelled wire is an adequate solution for coils with ratings up to several hundred volts. Test results confirm this with double-enamelled #18 (18-gauge) wire at 10 MHz.

Additional voltage breakdown strength can be had by providing greater separation at crossings as, for instance, by looping one crossing conductor over the others. Careful examination of the sketches in Fig. 2 will show that the right-hand side of each turn is behind the overlapping left-

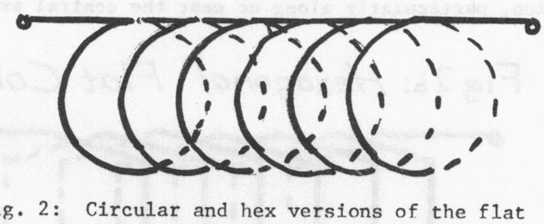


Fig. 2: a: Circular and hex versions of the flat coil. Note that the right-hand half of each turn is behind the left.

hand sides. (This will be true of any shaped turn.) Thus further breakdown strength can be had by simple expedients: the ends of each turn can be twisted slightly so that the underside of the top and bottom crossing is separated slightly from the corresponding top side. Or the turns can be tilted slightly out of the plane. This fact also suggests the possibility of inserting an insulating spacer along the long axis to separate front and back conductors. One hexagonal coil in Fig. 1 is so built. The PPI in Fig. 3 has the left-hand sides of its turns on the front of a double-clad board and the right-hand sides on the back.

In the case of PPI's voltage capability is influenced by both the surface insulating qualities of the board and its resistance to punch-through voltages. For WPI's which are flat-mounted on insulating sheet, the surface resistance will also be important.

3. Theoretical Explanation of Flat Coil Effect.

Since it is often supposed that a large part of the inductive effect of a helix is caused by the z-axis field common to many turns, the completely comparable inductance of flat coils -- either WPI's or PPI's -- is somewhat surprising. A flat configuration destroys any significant z-axis field and produces instead a field normal to the plane of the coil as sketched in Fig. 4. However, for coils with the same number of turns, turn size, and spacing, the inductance of flat coils and helical coils is the same within a few per cent.

The fact is the common z-axis field of an air core helix is largely a myth. Fig. 5 illustrates this point for the helix. The single turn marked T-T carries unit current and produces an H-field as sketched, strong near the conductor, weaker in the center of the current-carrying turn, and rapidly diminishing as we move away from T-T in any direction, particularly along or near the central axis r.

Fig 2b: Hexagonal Flat Coil

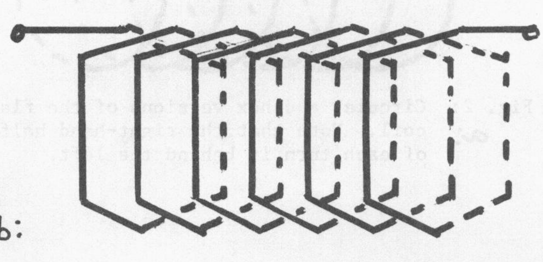


Fig 2b:

Now suppose that a second turn of the same radius is moved back and forth along the r-axis so that we can investigate its coupling with the current-carrying turn. Coupling can be represented by the fraction of the total flux passing through T-T which also passes through the second turn. At a-a the coupling is seen to be quite good (though by no means 100%). But at b-b, hardly more than a diameter removed, the coupling is very small. With most of the flux of T-T being concentric to and near the conductor itself, linkages are substantially limited to adjacent or nearly adjacent turns.

In calculating the total inductance of a coil it can be thought of as made up of the inductance of each turn plus the mutual inductance of each turn with every other. Thus,

$$L = nL_{\text{one turn}} + 2((n-1)M_{12} + (n-2)M_{1-3} + \ldots + M_{1,n}). \qquad [\text{Eq } 1]$$

For the helix, Fig. 5 suggests that the mutual inductance terms after the first few become quickly negligible.

Turning now to the flat coil we go through the same thinking. Fig. 6 examines the field in the plane of the unit-current-carrying loop on the left. A second loop of the same radius is slid along the r-axis. When superposed (zero offset) it theoretically links 100% of the first loop's flux. As it moves to position x it links less, and at y the linkage is greatly reduced and has, in fact, gone negative.

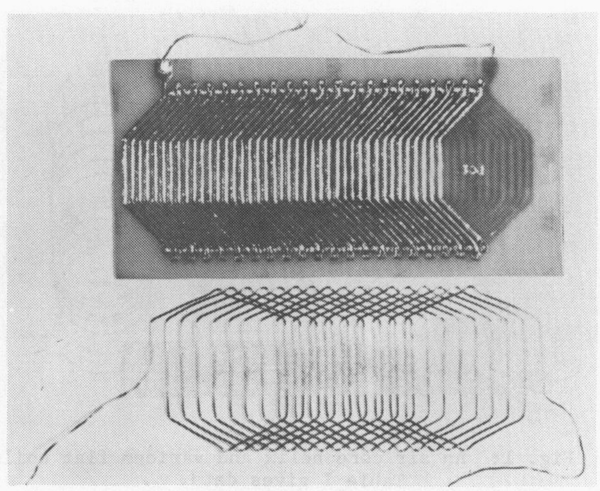


Fig. 3: A PPI coil made on double clad board. Note conductors on back showing through translucent board.

Thus it is not unreasonable to expect coupling between adjacent or nearly adjacent turns of the flat coil to be similar to that between closely oriented turns of a helix. The results of measurement and calculation given below confirm this.

An interesting difference between the two coils is this: The mutual inductance between remotely coupled turns of a helix, while small, is positive and augments slightly the total inductance. But for the flat coil, outside the current-carrying loop the field reverses so that remotely-coupled turns have a negative mutual inductance which slightly subtracts from the coil's total inductance. The fact that the two kinds of coils have closely comparable inductances is another evidence of the weakness of fields remote from a current-carrying turn.

A corollary of this reasoning is that as the second turn of the flat coil is slid away from superposition along the r-axis, some point must be reached where there is zero coupling. The computer programs to be discussed next show that for circular turns that point occurs at about 1.54 radii. Note also that the self inductance for one turn is the mutual inductance between two superposed turns (neglecting any strand thickness).

4. Calculations of Inductance and Experimental Results

The configuration of the flat coil as sketched in Fig. 2, being limited to a single plane, suggests the application of the Biot-Savart law to calculate inductance directly. The senior author and his students have made such calculations with three Fortran programs (discussed further in Appendix) organized as follows:

Program I applies the Biot-Savart law (Fig. 3),

$$H = \oint \frac{I \, dl \, \sin \theta}{4 \, pi \, D^2} \qquad [Eq\ 2]$$

to determine the H-field at any point on the r-axis. This involves a single integration around the current-carrying loop.

Program II (Fig. 6) takes the H-field results of Program I and calculates flux linkages (mutual inductance) in the second loop for any desired offset from the current-carrying loop. This calculation involves double integration over the surface of the second circle.

Program III takes as input mutual inductances calculated by Program II and implements Equation 1, above, to find the total inductance of a given flat coil.

Programs I and II are operated for unit radius (1 meter) and unit current. The H-field is calculated for every 100th of a radius along the r-axis, and the mutual inductance for two turns for offsets in tenths of a radius. Program III will combine and modify these results for coils of any radius. The handling of strand size, which is not particularly troublesome in practical cases, is discussed in the Appendix. These programs can be quickly modified for other radius gradations or even for paired loops of different diameters.

For calculating inductances of flat coils with non-circular turn shapes, circles of equivalent area have been substituted. The magnitude of the error thus introduced has not been investigated. Terman[2] suggests that as the polygonal-shaped turns approach a circle this error is minimized.

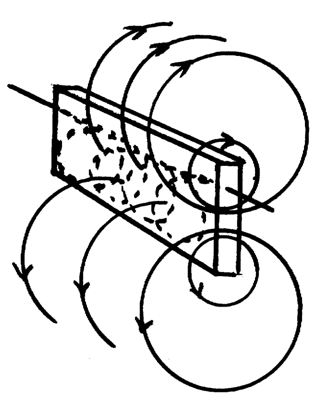


Fig. 4: The field of a flat coil is normal to its plane and tends to ring the long edges. In addition there are end effects.

TABLE II

Coil	Computer	Measured	Equation 3
5-turn circ WPI		2.9 uH	
12-turn hex WPI	4.9 μH	3.7	4.0 uH
13-turn circ WPI	5.0	5.8	3.9
17-turn hex PPI		10.7	9.5
17-turn hex WPI	8.5	8.3	6.9

Some comparisons of Inductances for Flat Coils by Computer Program, by Measurement, and by Formula of Reference #3.

The authors have made comparisons of inductance of both WPI's and PPI's 1) as measured in the laboratory with a Marconi model TF-1245 Q-meter, 2) as calculated with the three programs described, and 3) as approximated with the common formula for single-layer helixes,[3]

$$L\ (\mathrm{uH}) = \frac{n^2\ R^2}{9R + 10\ b}, \qquad [\mathrm{Eq}\ 3]$$

where R is the radius and b the length (both in inches) and n the number of turns. According to Terman equation 3 is good to about one per cent if b is greater than 80% of R.

Table II lists some of these comparisons which include output from Table III.

Table III lists some selected results from Program I, the H-field at different points on the r-axis.

Table IV lists some selected results of Program II, the mutual inductance of two loops for various offsets.

A confirmation of the reasonability of the assumptions in these programs and the fineness of the single and double integrations comes from calculating with them the inductance (zero offset) of a unit radius coil and comparing it to the single loop expression given by Terman[4]. A conductor diameter of 3% of the radius was assumed for both. Program calculated value of 5.44 uH. The Terman expression gives 5.46 uH.

5. Losses in Flat Coils.

Losses in these coils were measured by observing the Q's developed on the Marconi Q-meter during inductance measurements. Q's appear slightly poorer and a little more erratic in the WPI coils than for their helical counterparts. Measured Q's for various flat coils are given in Table I.

For the typical hf coils investigated made with conductor wire between #14 and #20, Q's for flat coils ranged from 125 to 320. Q ratios between flat and roughly comparable helical coils were 370/320 and 420/320.

In PPI coils, described below, measured Q's were lower, ranging from about 40 to 150 for inductances varying between 20 and 0.2 uH and at frequencies ranging from 1.8 to 58 MHz. The lower Q's in PPI's have not been adequately analyzed, but is presumably related to skin and proximity effects on the wide, shallow conductors and to losses in the board itself. The highest Q's occurred for the smaller coils in the VHF range. As frequencies are increased and inductances correspondingly reduced it becomes more and more practical to design flat coils with large cross-sectional conductors and minimized distributed capacitance to improve quality factor.

6. Etched Coils (PPI's)

Flat coils can be constructed by etching clad board in various ways. Such a coil is designated PPI: Printed Planar Inductor. Fig. 3 shows a PPI made from double clad board with the left-hand half of each turn on the top of the board and the right-hand half underneath. Because of backlighting in the photograph the parts of the coil on the back can be dimly seen. Connections between the front and back halves of each loop are made by soldering short lengths of wire through the board to pads on each side.

Other methods of etched coil construction which suggest themselves are bridging the top and bottom set of crossing for each turn with wire jumpers over the top of a single-sided board, bridging them similarly behind a single-sided

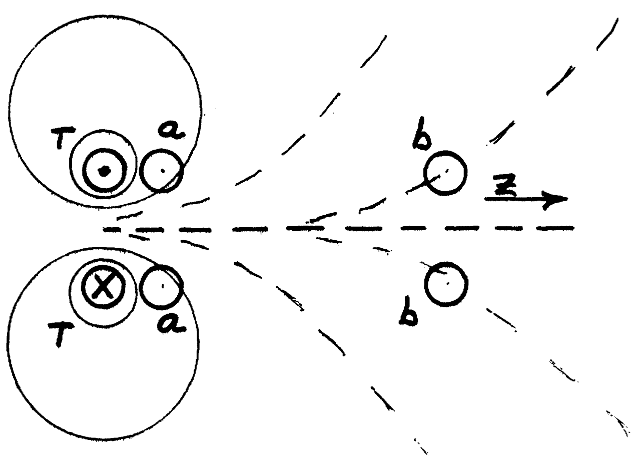


Fig. 5: The turns of a helix have little coupling unless they are adjacent or nearly so.

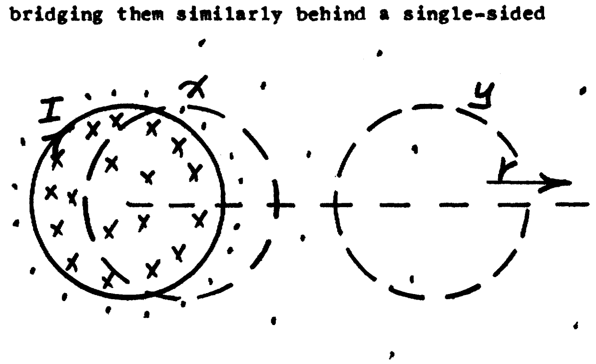


Fig. 6: Flat coil coupling is similar to a helix except that for remote turns the sign reverses.

board, or painting or otherwise applying insulating material to one set of conductors at these crossing areas and applying over this layer the bridging strand.

Distributed capacitances for PPI's are similar to those for WPI's.

A very attractive possibility with PPI's is arranging a computer program to do the art work for photographic transfer to the clad boards. In principle the coil user could decide on the inductance and dimensional and other parameters of the coil he wanted to produce, feed these variables into an established computer program, and take as output a plotted strand-pattern which would precisely produce the parameters desired. This would then be used photographically in the etching process as in ordinary printed circuit manufacture.

Fig. 7 shows examples of this art work produced on a Calcomp model 563 plotter from programs run on a DEC System-10 computer. Inputs to the program were: width of strand, minimum insulating surface width between strands, length of one side of the regular hexagon, number of turns, length of solder pads desired, mode of operation (stencil or filled), photoreduction factor to be used. It was found that with sufficiently narrow pens, filling in the entire width of a strand was tedious in operation. As an alternative, a "stencil" mode was also provided in the program which was quicker for trials but required the operator to fill in the outline by hand, an easy task.

7. Distributed Capacitance.

One disadvantage of flat coils is that their distributed capacitance, while reasonable for most applications, is larger than for helixes of similar inductance. Some C_d's are given in Table I.

TABLE I

Coil	R	l	Q	C_d	Strand	L	f
11-turn helix	2.5*	3.2 cm	360	7 pF	#14	5.6 UH	8 MHz
13-turn circ WPI	2.5	13.2	290	6	#19	5.8	8
11-turn hex WPI	2.6*	13.2	135	7	#20	3.7	8
16-turn hex WPI	2.8*	15.5	185		#20	8.5	4
34-turn hex PPI	2.2*	16.2	83	10		7.0	2.2

Some data on coils pictured in Figs. 1 and 3. Radii marked with an asterisk are calculated from equivalent area circular turns.

For two roughly comparable 5-turn coils at 8 MHz, one a WPI and the other a helix, the helix had a C_d of about 2 pF while the WPI had C_d of about 7 pF.

8. Coupling Between Flat Coils.

When two flat coils of the same size are placed directly on top of each other, the coefficient of coupling is measured to be about 70 to 80%. As a specific example the following data will illustrate this point and show how the coupling changes with displacement.

Two closely identical coils with circular turns (one is illustrated in Fig. 1) were connected in series. Each had 5 uH inductance when corrected for C_d. Each had 14 5-cm-diameter turns with average spacing of about 0.4 radii. They were placed directly on top of each other with the mutual inductance adding. They were then slid apart first in the longitudinal direction and then normal to the long axis. Finally they were separated along an axis normal to the planes of the coils. Detailed results for the variation in coupling for the three cases is given in Table V. In summary, along the r-axis coupling varied from 82% for zero displacement to negligible for full length longitudinal displacement, to about -20% for 1.6 radii lateral displacement, to about +10% for 1.6 radii displacement normal to the plane.

From these data it would appear that variable inductances could be made from two coils in series with ratios up to 5 or 10 times depending on how complex a mechanical arrangement is allowed in design. Considerable variation is possible with relatively small movement in the plane of the coils normal to the long axis. Other methods of varying the inductance to a lesser extent could include moving a conducting vane near the coils to lower inductance. This has yet to be investigated.

Because of the flat configuration it should be relatively easy to shield the two coils from each other electrostatically while still retaining good magnetic coupling.

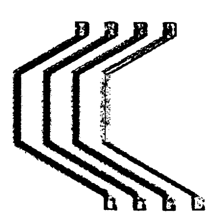


Fig 7: Computer-produced art work for PPI production

9. Advantages and Future Possibilities of Flat Coils.

The advantages of flat coils are, generally speaking, greatly reduced bulk, increased mechanical and electrical stability, and increased precision and reproducibility.

Disadvantages include increased distributed capacitance and somewhat increased losses. For high power transmitting coils special design care must be taken to avoid voltage breakdown.

The bulk reduction for some applications may be limited by the need to provide adequate space for the magnetic field. But even where this requirement is severely restrictive it appears that a two-to-one bulk reduction is available. In most cases a bulk reduction of five to ten is indicated.

WPI's can be supported in whole or in part as desired so that suppression of vibration and dimensional change is limited only by the supporting insulating materials. These coils are easily shielded magnetically and electrostatically.

The precision and reproducibility of coil design and manufacture may eliminate the need for some adjustable coils -- for example in the fixed rf amplifier circuits of tuners. Further it is possible to visualize one-board systems in which coils for multiple channels are printed or affixed flat, with provisions for the introduction of active elements, switches, fixed and variable capacitors, along with IC's for control.

TABLE III

r in radii	H(r) in NI/m
0.0	.50
.1	.50
.2	.52
.4	.57
.6	.71
.8	1.13
.9	1.96
.95	3.60
.98	8.43
.99	0.00
1.00	0.00
1.01	0.00
1.02	-7.49
1.1	-1.26
1.5	- .14
2.0	- .04
3.0	-0.01
5.0	-0.002

Field in plane of unit-radius-circular-turn carrying unit current.

No substantial investigation has yet been made on combining these flat coils with core materials. The field sketch of Fig. 4 suggests that laminated core material be applied in planes normal to the long axis in two sets, one above the median line and the other below.

The simplicity and extensibility of the field pattern -- always normal to the plane of the coil -- suggest the use of the flat coil to produce tailored field patterns. The field strength is easily adjusted by changing winding pitch. Further, on flexible sheets of supporting insulating material, these flat field patterns could be twisted into three-dimensional configurations as, for example, on the neck of a CRT.

10. Appendix -- Further Discussion of Computer Calculation

An interesting problem in calculating inductance is how to make allowance for the diameter of the wire. The self inductance of a single turn is significantly affected by conductor size. Using Terman's single turn expression[4] one obtains a variation of about 30% in inductance of a 2-meter turn for wire size variation from 0.03 radii to #18 gauge. While 3% of the radius may seem large for a unit circle, it represents typical coil practice at smaller diameters and so permits Program III to scale the results for small radii. Neglecting skin effect, the inductance of a given shape coil is proportional to one dimension.

The mutual inductance between two turns is little affected by conductor size. Thus equation 1 is susceptible to considerable variation in the self inductance terms but is relatively stable for mutual inductance terms.

A common practice in calculating inductance is to neglect any field inside conductors, which probably corresponds pretty well with practical situations, particularly at higher frequencies. In the computer

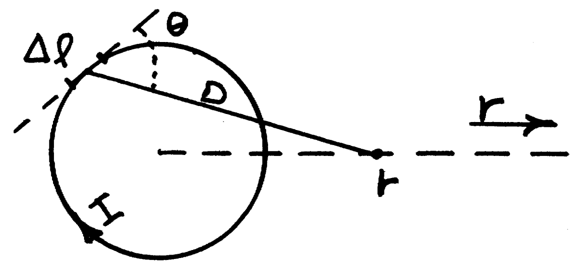


Fig 8: Applying the Biot-Savart Law to flat coil field computing.

calculations described in section 4 of this paper, it is easy to modify the results of Program I for wire size by setting the H-field values output to zero for values of r which fall inside the current-carrying conductor. This was done on the basis of a 3% of R conductor diameter with the excellent results described in the last paragraph of section 4 above.

The single integration of Program I around the current-carrying loop (implementing the Biot-Savart formula) was carried out with delta-l's of 200 to the full circle. The double (area) integration of Program II (finding flux linkages) was carried out on a polar coordinate basis with delta-angle of 200 parts to 2 pi radians, and delta-radius of 100 parts. For both these programs at locations beyond a certain distance from the current-carrying loop, there is no reason to retain this degree of fineness for machine integration. This point has yet to be investigated.

The principle assumptions in the programs were: 1) that the coil turns were perfectly circular, which is not quite true, and 2) the procedure described above for neglecting field inside a conductor. The senior author will be glad to furnish copies of these programs to anyone interested.

11. Acknowledgements.

The student authors' principle work was divided as follows: M. Allyn -- voltage breakdown; R. Bradley and J. Gabranski -- computer inductance programs; W. Gist and J. Roberge -- built and tested PPI's; C. Gerstle -- computer-programmed art for PPI's; T. Graves -- measurements; S. Kay -- measurements & coupling.

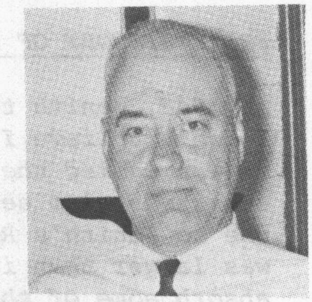
TABLE IV

Offset in Radii	M in uH
0.0	6.73
0.5	1.74
1.0	.76
1.5	.04
2.0	- .52
2.5	- .10
3.0	- .05
3.5	- .03

Coupling of flat coil turns of unit radius with offset.

12. References.

1. The flat coil (WPI and PPI) is the subject of U.S. Patent Number 3,717,835.
2. F. E. Terman, "Radio Engineers' Handbook," 1943, McGraw-Hill, p 56.
3. idem p 55.
4. idem p 52, eq (29).



13. Biography.

W. H. Roadstrum took his BSEE at Lehigh and his MS and PHD at Carnegie-Mellon. He has served as an engineer at the Pittsburgh Station of the U.S. Bureau of Mines, and as an engineer and manager for General Electric in Ithaca, Utica, and Schenectady. He has taught at the Naval Postgraduate School in Annapolis and Monterey and is currently a professor of electrical engineering at WPI in Worcester, Massachusetts. He has served his local IEEE section in a number of posts including chairman.

TABLE V

r-axis offset - % length $L_1=L$

r-axis offset (% length)	$L_1+L_2\pm 2M$ (uH)
0	18.2
23	16.0
46	13.0
77	10.6
100	10.4

lateral offset (radii)	$L_1+L_2\pm 2M$ (uH)
0	18.2
.2	17.3
.4	15.7
.6	14.0
.8	12.0
1.0	11.0
1.2	9.4
1.4	8.4

normal separation of planes (radii)	$L_1+L_2\pm 2M$ (uH)
0	18.2
.24	17.3
.48	13.5
.80	12.8
1.28	11.9
1.64	10.9

Variation in coupling of 2 5-henry circular WPI's in series for separation along three axes.